Abstract

This paper applies the methodology developed by Forte and Peña (2006) to extract the implied default point in the premium on credit default swaps (CDS). As well as considering a more extensive international sample of corporations (96 US, European and Japanese companies) and a longer time interval (2001-2004), we make two significant contributions to the original methodology. First, we calibrate bankruptcy costs, allowing for the adjustment of the mean recovery rate of each sector to its historical average. Second, and drawing on the sample of default point indicators for each company-year obtained, we propose an econometric model for these indicators that excludes any reference to the credit derivatives market. With this model it is thus possible to estimate the default barrier resorting solely to the equity market. Compared with other alternatives for setting the default point in the absence of CDS (such as the optimal default point for shareholders, the default point in the Moody’s-KMV model or the face value of the debt), the out-of-sample use of the econometric model significantly improves the capacity of the structural model proposed by Forte and Peña (2006) to differentiate between companies with an investment grade rating (CDS less than 150 bp) and those with a non-investment grade rating.

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Introduction

A central element in structural credit risk models is the definition of a stochastic process for the total value of the company's assets, and the assumption that default arises when its value reaches a specific minimal threshold. This threshold is usually referred to as the default barrier or the default point.¹

The empirical testing of these models usually involves analysing their capacity to generate credit risk premia that are consistent with those observed in other markets such as the debt market. Such testing is normally hampered by the fact that a good number of the parameters common to most structural models are not directly observable. This is the case, for instance, of the volatility of the total value of assets, of the potential costs of bankruptcy, and of the default point.² As a result, researchers need to define a procedure for estimating the parameters, which ultimately means testing turns into a simultaneous test on the model and on the estimation procedure.

In a recent paper, Forte and Peña (2006) (FP hereafter) propose a structural model consisting of a modified version of the well-known model of Leland and Toft (1996). One distinguishing aspect of the FP paper is that, along with the model, it proposes a specific procedure for determining the parameters. The methodology proposed (theoretical model and estimation procedure) has two fundamental characteristics. Firstly, it considers that bankruptcy costs may be assumed to be equal to zero when valuing the company's total assets, although such costs are clearly relevant with a view to valuing debt. The reason lies in the fact that such costs do not affect the total value of the assets, solely the percentage of assets that will remain in

¹ The possibility of default is restricted on occasions to specific periods. In the seminal paper by Merton (1974), for example, default can only take place on maturity of the debt. It is moreover habitual in this type of model to speak interchangeably about default and bankruptcy, although both events need not be associated. This interchangeability will also be applicable in this paper.
² Although the default point is not observable, many models provide a guide to determine it. This is the case, for instance, of models with an endogenous default point.
creditors' hands in the event of bankruptcy. In this respect, FP adopt an approach similar to that proposed by Goldstein, Ju and Leland (2001). The second key aspect is the termination of the default point. FP propose calibrating this parameter on the basis of the information available in other markets, particularly in the CDS market. This procedure is analogous to determining the volatility of shares on the basis of the price of their associated options.

FP demonstrate that the methodology described allows, for most companies, for the generation of credit risk premia drawing on stock market capitalisation and an a small number of accounting items, which would be in line with those observed in the bond or CDS markets. In this way the authors are able to analyse, on the basis of a uniform measure such as the credit risk premium, the different speed with which the three markets (for bonds, CDS and equities) incorporate new information in respect of credit risk. FP conclude that the equity market leads the other markets when it comes to incorporating this information, with no clear pattern of leadership between the bond and CDS markets.

One limitation of the procedure proposed by FP is that it is not applicable to companies without CDS or reasonably liquid bonds, and it is precisely in these cases when the information that can be generated from the equity market will prove more valuable. In this paper we analyse the determination of the default point when the only market information available is that provided by the equity market. To do this we consider a broader international sample of companies than in FP (96 US, European and Japanese companies), and one which spans a longer period (2001–2004). We apply the methodology described to the sample with a view to obtaining the default point indicator (the ratio between the default barrier and the face value of the total debt) for each company-year observation. One fundamental contribution compared with the paper by FP is that instead of considering exogenous bankruptcy costs like these authors, such costs are calibrated on the basis of the sector in question. The aim is that
once the process to estimate the default point indicators is over, not only will the premia observed in the CDS market be replicated, but an expected recovery rate will be obtained for each sector adjusted to the historical evidence. The main conclusion from applying this procedure is that the bankruptcy costs would on average be around 60% of the value of the company’s assets, far above what is traditionally assumed by the literature.\(^3\) In the wake of these results, these costs should be broadly interpreted and include, in addition to legal costs, the loss of future income incurred by the company owing to the potential discontinuation of operations.

Based on the series of premia in the equity market and in the CDS market, we perform an analysis of price discovery to provide further evidence on this process. In line with the results obtained by FP, we find that the equity market leads the CDS market in the incorporation of new information on credit risk. This conclusion is valid for all the periods (2001–2004) and economic areas (United States, the euro area and Japan) considered.

Hereafter, and drawing on the sample obtained of default point indicators further to calibration with the CDS market, an econometric model is developed for these indicators. The model is capable of representing up to 84% of the variability in the default point indicator using a very small number of explanatory variables. Any reference to the CDS market is excluded from these variables, and the model is thus susceptible to being applied to companies for which data on this market are not available.

For the following step we estimate the default point indicator for each company-year of the sample based on the econometric model, and we re-calculate the equity market premia series using these new indicators. We find that although the estimator of the default point indicator is unbiased outside the sample (it leads on average to the same value suggested by the CDS), the high sensitivity of the

\(^3\) Both Forte and Peña (2006) and Leland (2004) assume a value of 30%.
estimated premium in respect of this parameter may cause significant deviations from the premium observed in the CDS market, especially when what is obtained is an overestimation of the default point. Using the model, however, allows companies to be classified in different levels of credit risk with greater accuracy than with other procedures. This is the case, for instance, when what is sought is a distinction between companies with an investment grade rating and companies with a non-investment grade rating. Specifically, given the null hypothesis that a company's CDS is below 150 bp for a specific date (which is equivalent to showing an investment grade rating), the use of the indicators generated by the econometric model is, among all the possible alternatives to direct calibration with CDS, the procedure which, maintaining the level of significance below 10%, offers most testing power (69% compared with 30% for the best alternative consisting of use of the theoretical optimal point).

The rest of the paper is structured as follows. Section I reviews the FP methodology. Section II analyses the sample of companies and implementation of the procedure described in Section I. Section III studies the process of price discovery. Section IV develops the econometric model, while Section V tests its usefulness with a view to a potential out-of-sample application. Finally, Section VI draws the main conclusions.

**The Forte and Peña (2006) methodology**

The FP methodology is essentially a modification of the Leland and Toft (1996) model, as well as a procedure for estimating the default barrier based on information on the credit risk premium in markets other than the equity market, and in particular in the CDS market.

The original Leland and Toft (1996) model has as its premise that the total value of the company’s assets, $V$, moves according to the following continuous diffusion process
where $\mu$ and $\sigma$ represent the expected return of $V$ and its volatility, respectively, $\delta$ the proportion of the total value of the assets set aside for payment to investors (interest and dividends), and $z$ describes a standard Brownian process. Under these assumptions, Leland and Toft (1996) show that the value at any point in time $t$ of a bond with a maturity $\tau$, principal $p(\tau)$, coupon $c(\tau)$, and whose holder receives a fraction $\rho(\tau)$ of the value of the assets in the event of default, will be given by the following expression

$$ d(V, \tau, t) = \frac{c(\tau)}{r} + e^{-\tau} \left[ p(\tau) - \frac{c(\tau)}{r} \right] \left[ 1 - F(\tau) \right] + \left[ \rho(\tau) V_B - \frac{c(\tau)}{r} \right] G(\tau) $$

where $r$ is the risk-free rate and $V_B$ the default barrier. The expressions $F(\tau)$ and $G(\tau)$ will in turn be given by

$$ F(\tau) = N[h_1(\tau)] + \left( \frac{V}{V_B} \right)^{-a} N[h_2(\tau)] $$

$$ G(\tau) = \left( \frac{V}{V_B} \right)^{-a+z} N[q_1(\tau)] + \left( \frac{V}{V_B} \right)^{-a-z} N[q_2(\tau)] $$

with

$$ q_1(\tau) = -\frac{b - z \sigma^2 \tau}{\sigma \sqrt{\tau}}; \quad q_2(\tau) = -\frac{b + z \sigma^2 \tau}{\sigma \sqrt{\tau}} $$

$$ h_1(\tau) = -\frac{b - a \sigma^2 \tau}{\sigma \sqrt{\tau}}; \quad h_2(\tau) = -\frac{b + a \sigma^2 \tau}{\sigma \sqrt{\tau}} $$

$$ a = \frac{r - \delta - \sigma^2 / 2}{\sigma^2}; \quad b = \ln \left( \frac{V}{V_B} \right); \quad z = \left[ (a \sigma^2)^2 + 2r \sigma^2 \right]^{1/2} $$
On the basis of equation (2), FP suggest expressing the default point \( V_b \) as a fraction \( \beta \) of the face value of the total debt \( P \). Assuming then that each creditor receives, in the event of default, a fraction of that value (net of bankruptcy costs) proportionate to the weight of the face value of their debt relative to the total debt, that gives the following alternative expression for the value of the bond 4

\[
d(V, \tau, t) = \frac{c(\tau)}{r} + e^{-\tau r} \left[ p(\tau) - \frac{c(\tau)}{r} \right] \left[ 1 - F(\tau) \right] + \left[ (1 - \alpha) \beta p(\tau) - \frac{c(\tau)}{r} \right] G(\tau)
\]  

(3)

where \( \alpha \in [0,1] \) represents the bankruptcy costs.

The total value of the debt, \( D(V, t) \), will be the sum of the value of all individual bonds. If we assume N bonds have been issued, and if we denote the face value of the \( i \)th bond as \( \tau_i \), then

\[
D(V, t) = \sum_{i=1}^{N} d(V, \tau_i, t)
\]  

(4)

Another fundamental equation in the FP methodology is that which relates the value of own capital, \( S(V, t) \), to the value of the firm’s total assets

\[
S(V, t) = V(t) - D(V, t | \alpha = 0)
\]  

(5)

where \( D(V, t | \alpha = 0) \) represents the value of the debt under the assumption that the bankruptcy costs are equal to zero. The intuition whereby \( \alpha \) does not form part of the equation defining \( S(V, t) \), although it does affect the valuation of the debt in keeping with expressions (3) and (4), is based on the fact that the shareholders will not be directly affected by the firm's loss of value in the event of bankruptcy, since the creditors are the only parties that bear this cost. 5

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4 See also Leland (2004).
5 For a discussion of this point see Forte and Peña (2006).
**Calibration procedure**

The credit risk premium provided by the structural model at each point in time $t$ is determined as the theoretical premium of issuing a bond at par with a maturity equal to that of the CDS that are to be used subsequently for the calibration of $\beta$, and which we will assume to be equal to 5 years. Specifically, this bond should pay a coupon such that the following condition holds

$$d(V,5,t|p) = p$$

(6a)

If we denote this coupon as $c_t(5,p)$, the bond yield will be

$$y_t^5(5) = \frac{c_t(5,p)}{p}$$

(6b)

whereupon the premium obtained on the basis of the structural model will respond to the differential between this yield and the risk-free rate

$$ICS_t = y_t^5(5) - r$$

(6c)

To apply this procedure it is necessary to have at each point in time $t$ information on:

I.1. Value of the company $V_t$.
I.2. Face value of total debt $P_t$.
I.3. Risk-free rate $r_t$.
I.4. Payout $\delta_t$.
I.5. Volatility $\sigma_t$.
I.6. Bankruptcy costs $\alpha_t$.
I.7. Default point indicator $\beta_t$.

FP propose considering volatility, bankruptcy costs and the default point indicator as constant, and allowing the rest of the variables to
depend on t. In order to determine these variables, the following data will firstly be collected:

D1. Daily data on stock market capitalisation.

As we will see below, the estimation of the total value of the assets based on stock market capitalisation in keeping with expression (5), is the key factor that allows the information provided by the equity market in terms of credit risk premia to be translated.

D2. Accounting data referring to

D.2.1. Short-term liabilities (STL).
D.2.1. Long-term liabilities (LTL).
D.2.3. Interest payments (IE).
D.2.4. Dividend payments (CD).

These accounting data will typically be available quarterly, half-yearly or annually, whereby some type of interpolation is proposed in order to translate them into daily data.

Total liabilities \( (TL) \) will be the sum of short- and long-term liabilities. That gives

\[
P_t = TL_t; \quad t = 1, ..., T
\]  \hspace{1cm} (7)

The payout \( \delta_t \) shall be expressed as

\[
\delta_t = \frac{CD_t + IE_t}{V_t}; \quad t = 1, ..., T
\]  \hspace{1cm} (8)

Having thus assumed a value for \( \beta \), the series of the total value of the assets \( V_t \) can be estimated, as can volatility \( \sigma \), by means of the following algorithm
1) Proposing an initial value for $\sigma, \sigma_0$.
2) Taking as a basis the observed stock market capitalisation series, $S_t$, estimating the series $V_t$ so that the relationship expressed in (5) holds for all $t$.
3) Estimating the volatility of $V_t, \sigma_t$, on the basis of the series obtained in 2).
4) Concluding whether $\sigma_1 = \sigma_0$. Otherwise proposing $\sigma_1$ in step 1 and repeating until convergence.

However, this procedure calls for the total value of the debt to be determined when bankruptcy costs are zero, $D(V, t | \alpha = 0)$. On the basis of equation (4), and imposing $\alpha = 0$, it is possible to express this value as the sum of individual bonds. It therefore becomes necessary to interpret the information available on the debt (short- and long-term liabilities, and interest payments) in the form of such bonds. FP suggest considering that the company maintains a total of ten; one with the face value of the short-term debt and with a maturity equal to one year, and nine with a maturity of 2 to 10 years respectively, and a face value for each one equal to 1/9 of the long-term debt. Furthermore, a coupon is assigned to each of these 10 bonds, representing a fraction of the annual payment of interests proportionate to the weight of the face value of the bond relative to the face value of the total debt.

The risk-free interest rate to be applied to each of these bonds will be the swap rate corresponding to their maturity. The following information will thus have to be reflected:

D.3. Daily data on the 1-10 year swap rate, i.e. $r_t^s(\tau); \tau = 1, ..., 10$.

which at the same time provides the rate to be applied in (5)

$$r_t = r_t^s(5); \ t = 1, ..., T$$ (9)
Regarding the bankruptcy costs, FP propose following previous papers (Leland, 2004) using a fixed value for all the companies, specifically

\[ \alpha = 0.3 \]  \hspace{1cm} (10)

whereby the sole parameter to be determined is \( \beta \).

The following relationship between the series \( ICS \) and \( CDS \) is then assumed

\[ ICS_i = CDS_i \times e^{\varepsilon_i} \]  \hspace{1cm} (11)

where the \( \varepsilon_i \) are errors \( i.i.d. \) with \( E[\varepsilon_i] = 0 \) \( \text{y} \) \( \text{Var} (\varepsilon_i) = \sigma^2_\varepsilon \). Under these conditions the Mean Squared Error is

\[
MSE = \frac{1}{T} \sum_{t=1}^{T} \left[ \log \left( \frac{ICS_t}{CDS_t} \right) \right]^2
\]  \hspace{1cm} (12)

and \( \beta \) is finally determined as that value of the default point indicator which minimises this measure of discrepancy between series, i.e.

\[
\beta \equiv \arg\min_{\beta} (MSE)
\]  \hspace{1cm} (13)

In sum, the implied credit risk premium in the equity market is constructed on the basis of (6). The necessary arguments, detailed in I.1 – I.7, are estimated using the data described in D.1 – D.3 and equations (6) – (13).
Data and Implementation

A. Data

The initial sample contains daily data on five-year CDS for 120 non-financial corporations in the United States (dollar-denominated CDS), Europe (belonging to the euro area and with euro-denominated CDS) and Japan (yen-denominated CDS). The data have been taken from Credit Trade, and are confined to the period from 2 January 2001 to 31 December 2004.

For the equity market, daily data on stock market capitalisation are obtained from DataStream. Taken from this database, also with a daily frequency, is the 1-10 year swap rate in dollars, euro and yen. The accounting items required by the FP methodology (short- and long-term liabilities, along with interest and dividend payments) are obtained from WorldScope.

B. Implementation

The general procedure described in Section I is specified in our case as follows:

1. The accounting data at each point in time $t$ are determined by linear interpolation among the annual data obtained drawing on WorldScope.

2. The sample is divided into calendar years, so that the $\beta$ are adjusted by year (unlike FP, which adjust them by half-year). No company-year observation is considered unless at least 150 observations of CDS for that year are available, and no company remains in the sample if it is not possible to consider at least 2 consecutive years.
A total of 7 companies are eliminated in accordance with the restrictions imposed in point 2, either because of a lack of sufficient data on CDS or because of the absence of the related data for the equity market.

FP indicate moreover that it is not advisable to apply their methodology to companies involved in mergers or acquisitions, and they analyse the specific example of the merger of Olivetti and Telecom Italia. The reason is that as the merger or acquisition draws near, the credit premiums for these companies will tend increasingly to represent the credit risk of the joint company, with the information on one of the companies involved proving insufficient in this case. The following point of the implementation is intended to eliminate those companies subject to merger or acquisition processes that may have a significant bearing on results:

3. For each company a study is made of whether, during the sample period, merger or acquisition operations are under way. If they are, the company is eliminated if such an operation entails an amount higher than 5% of the total value of its assets, and if it gives rise to a change in its capitalisation of more than 10%.

This procedure involves eliminating another 6 companies from the sample.

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6 The development of an adjustment allowing this methodology to be used on the consolidated theoretical company is, moreover, beyond the objectives of this study.
7 To identify merger and acquisition operations we have used the SDC Platinum database, which was made available to us thanks to Ricardo Gimeno.
8 We should acknowledge that such a procedure does not prevent the possibility of certain companies affected by mergers or acquisitions from continuing in the sample, owing to the fact that completion of the operation may have come about subsequently to our sample period. Foreseeably, however, such a possibility will not significantly influence the results.
4. For the companies that remain in the sample after step 3, an initial estimation (E1A) is made assuming a constant \( \beta \) for each company throughout the period, \( \beta_{01-04} \).

FP indicate that on this point it is possible to obtain two solutions: one with a "small" \( \beta \) and one with a "large" \( \beta \). They justify the first solution as being more reasonable, since the second one means values of \( \beta \) typically above 1, which does not generally appear rational from the standpoint of the shareholders.\(^9\) To ensure that the first solution is obtained, and adhering once again to FP, the following procedure is applied:

a) Choose an initial, sufficiently small value for \( \beta_{01-04} \), \( \beta_0 \) (specifically 0.3), and define \( \beta_i = \beta_0 + 0.05 \).

b) Evaluate \( MSE_0 = MSE(\beta_0) \) and \( MSE_1 = MSE(\beta_i) \).

c) If \( MSE_1 < MSE_0 \), define \( \beta_0 \) once again as \( \beta_0 = \beta_i \) and return to step a).

d) If \( MSE_1 \geq MSE_0 \), seek the value of \( \beta \) that minimises the MSE in the interval \( (\beta_0 - 0.05, \beta_0 + 0.05) \).

If the corner solution \( \beta_{01-04} = 0.3 \) is obtained, the procedure is repeated, reducing the initial value of \( \beta_0 \).

As in FP, each new proposal for \( \beta \) entails a new estimation of volatility in accordance with the algorithm described in Section I. In this connection, all the accounting and capitalisation data available are used (from 2 January 2001 to 31 December 2004).

5. Taking \( \beta_{01-04} \) of the initial proposal for each company, a new estimation (E1B) is made in which \( \beta \) is allowed to

\(^9\) We may add that it is not consistent either with the empirical evidence on recovery rates.
vary from year to year. This gives rise to a vector \( \beta_s = (\beta_{01}, \beta_{02}, \beta_{03}, \beta_0) \) for each company.

Each proposal for a new vector entails a new estimate of volatility, where in this case the links between years are omitted so that the changes in \( \beta \) do not affect the findings. In those cases in which there are not sufficient CDS data for a year, the value of \( \beta \) closest in time is applied when estimating this volatility.

At this point it is worth referring to the choice of bankruptcy costs, \( \alpha \), and their consequences for the estimation of \( \beta \). The parameter \( \beta \) determines the default point, and therefore the probability of default for a specific time horizon, both the actual probability and the risk-neutral probability. But this parameter also determines, along with \( \alpha \), the rate of recovery in the event of default \( (1-\alpha)\beta \). Just as Leland (2004) justifies the fact that different values of \( \alpha \) and \( \beta \) can replicate a single recovery rate (RR), but only one of these pairs of values is capable of replicating moreover a specific (real) probability of default, we have it in our case that the values of \( \alpha \) and \( \beta \) capable of replicating a specific RR are infinite, but only one of those pairs allows, moreover, a specific credit risk premium to be replicated. Indeed, this premium depends on the RR and on the (risk-neutral) probability of default. Having set an objective RR drawing on the empirical evidence, there will only be one pair of values for \( \alpha \) and \( \beta \) that additionally allow the CDS and ICS series to be adjusted.

Assuming \( \beta \) equal to 0.731, Leland (2004) manages to replicate the expected default frequencies (EDF) of different credit ratings. This, combined with bankruptcy costs of 30%, enables an RR of around 51% to be reproduced at the same time. FP likewise assume bankruptcy costs of 30%. For their sample, the calibration of \( \beta \) gives rise to an average value of 0.792, and therefore to an average RR of 55%. Significantly, both papers arrive at very similar estimated values departing from different approaches. One seeks to replicate EDF and
the other premiums in the CDS market. Both Leland (2004) and FP consider that an RR slightly higher than 50% is reasonable given the historical evidence.

An RR of around 50% need not, however, be the most appropriate reference value for any sector or any period. Table I includes the average RR for defaults observed by sector in Varma, Cantor and Hamilton (Moody’s Special Comment, 2003), and for the period 1982-2003. What is primarily striking is the dispersion by sector, ranging from 23.2% in telecoms to 51.5% in the utility-gas sector. Secondly, the total average is 35.4%, far down on the figure of 50% or 55% that is usually considered representative. This reduction can be explained by the inclusion of recent years in which the average RR has been particularly low (25.6% in 2000 according to the same paper). Table I indicates, moreover, the sectors in which the RR has not been statistically different from the average, and those in which it has been statistically higher or lower than that average.

Different RR depending on the sector involved may undoubtedly be due to different average values for the $\beta$. However, it is reasonable to expect that bankruptcy costs will vary too depending on the sector. Habitually, these costs are identified with the direct charges associated with the legal resolution of a default, and where appropriate with settlement, whereby it is reasonable to assume that they depend especially on the legal framework established by the country in which the conflict is resolved, and to a lesser extent on the sector. Nonetheless, these legal costs appear to represent a small proportion of the company's loss of value in the event of bankruptcy (between 4% and 8% for developed economies according to the Global Financial Stability Report of the International Monetary Fund, 2004). The higher percentage of these costs might therefore be associated with the loss due to transfer of future income, such transfers being made more or less free of charge by the company to other firms in its sector, on deciding to discontinue operations. In the case of companies whose activity is linked to specific tangible assets, as is the case with gas
corporations, this free transfer will be minor, as any company wishing to take up the market share freed up by a bankrupt company will possibly have to purchase from that company a portion of its tangible assets. In the case of other types of sectors whose activity is linked to a lesser extent to specific assets, as is the case with telecoms, the transfer of income will be sizeable, since the companies in this sector may assume the market share freed up without compensating in any way the bankrupt company.

It thus seems reasonable to introduce sectoral variability into the bankruptcy costs. To this end, the $\beta$ estimation procedure in respect of that proposed by FP is extended as follows:

6. Those companies with an MSE higher than one after the EIB estimation are eliminated from the sample. The remaining companies are grouped according to the sectoral classification of Table I.\textsuperscript{10} Taking as a reference the value of $\beta_{01-04}$ obtained for each company in the EIA estimation, the average RR for the sector, given the assumed value for the bankruptcy costs ($\alpha = 0.3$), is calculated.

FP offer examples to argue that an "abnormally high" MSE may be indicative of the presence in the CDS of information other than the credit risk associated with the company's financial position. It is important to strip out these companies before calculating the average RR by sector owing to the potential bias that they might add to these averages. Up to 11 companies are eliminated for this reason.

7. In the light of the results of point 6, a selection is made in each sector of a representative firm as far as the RR is concerned, namely the firm whose RR is closest to the

\textsuperscript{10} Our classification matches the FTSE/JSE Global Classification System. With these groupings, therefore, we consider an equivalence between the ten sectors of this classification and those included in Table 1.
average obtained for its sector following the EIA estimation. The specification is then made of the bankruptcy costs it is necessary to assume so that the company, once a fresh estimation of $\beta_{01-04}$ has been made, may generate an RR equal to the historical average for its sector.

The use of a representative company allows, for each possible value of $\alpha$, $\beta_{01-04}$ to be re-estimated for a single company by sector. The alternative would be to make a re-estimation of $\beta_{01-04}$ for all the companies in each sector and to calculate in each case the new average, which would be very costly in computational terms. As we shall see, the proposed procedure suffices to give a reasonable approximation to the bankruptcy costs in terms of the sector.

8. The bankruptcy costs obtained in point 7 are imposed below on the rest of the companies depending on the sector, and points 4 and 5 are repeated, i.e. a new estimation is made assuming a constant $\beta$ (E2A), along with a subsequent estimate allowing it to vary from year to year (E2B).

The results of steps 6 to 8 are shown in Table II. Panel A reflects the results of assuming bankruptcy costs of 30% for all companies. Evidently, this leads to the systematic overestimation of the RR for all sectors, which would appear to indicate that such costs will be higher in all cases than the assumed figure of 30%. This is duly indicated by the results of steps 7 and 8 contained in panel B. The panel reflects how the method applied enables an RR by sector consistent with the empirical evidence to be generated, albeit at the expense of assuming bankruptcy costs far above those traditionally accepted by the literature.\footnote{In all cases the $MSE$ remains below 1 after the E2B estimate.} The most evident case in this respect is that of the telecoms sector, with estimated bankruptcy costs of 69%. It is
however difficult to reconcile bankruptcy costs of between 10% and 20% (as estimated by Andrade and Kaplan, 1998), or even of 30% (as assumed by Leland, 2004, and FP), with a loss given default (LDP) of 77% (alternatively, an RR of 23%). Bankruptcy costs of 30% would be consistent with an RR of 23% if a value for $\beta$ equal to 0.33 were assumed, i.e. if telecoms were capable on average of withstanding the total value of their assets being equal to 33% of the face value of their debt, without defaulting on the payment of such debt. The average value of $\beta$ obtained for these companies is, however, 0.74. Continuing with the previous arguments, we have it that both the combination $(\alpha = 0.3; \beta = 0.33)$ and the combination resulting from the estimation E2A $(\alpha = 0.69; \beta = 0.74)$ allow an envisaged average RR for this sector of close to 23%, but only the second combination gives rise, moreover, to credit risk premiums for the equity market consistent with those observed in the CDS market.

C. The Final Sample

As a result of the procedure described, the following data are obtained for a final sample of 96 companies:

a) Daily series of credit risk premia drawn from the CDS market ($CDS$).

b) Default point indicators in annual terms further to calibration with the CDS market ($\beta_{CDS}$).

c) Daily series of credit risk premia drawn from the equity market further to calibration with the CDS market ($ICS_{CDS}$).

Table III offers some descriptive statistics of these companies' CDS. As might be expected, an inverse relationship with the company's rating is observed. These premiums are also seen to be on a declining trend throughout the period 2002–2004 (the number of companies
with data for 2001 is relatively insignificant). Across the economic areas, the United States shows the highest average levels, followed by the euro area and finally by Japan.

Table IV contains the various measures of the differential between the ICS and CDS series habitually used in the literature. This differential is shown to be greater in absolute-value terms (avab) the worse the rating is, but highly stable in relative terms (avab(%)). Both results would be consistent with a log-linear relationship between the series in keeping with expression (11). The inverse relationship between credit rating and differential in absolute-value terms may moreover explain very well the results in terms of years and economic areas. Thus, the improvement in credit rating in our sample during the period 2002–2004 (represented by the decline in CDS) was accompanied by a reduction in this differential. Likewise, we find that the United States, the area with the biggest levels of CDS, is also the region with the biggest differentials in absolute terms. At the other extreme would be Japan, with the lowest levels of CDS and the lowest differentials. Generally, the discrepancy between series appears in our case greater than that obtained by FP (28.66% on average in relative terms for the entire sample, compared with 18.79% in the aforementioned study). It should however be recalled that, in their study, FP adjust the value of \( \beta \) on a half-yearly basis, while in our case this adjustment is made on a yearly basis. Evidently, the greater the frequency with which \( \beta \) is estimated, the better the adjustment will be.

Fig. 1 shows the distribution of the \( \beta_{c} \) (company-year observations), and the main descriptive statistics. The results range from a minimum of 0.18 to a maximum of 1.22, while the mean and median are around 0.85, slightly above the FP figure of 0.792. With a standard deviation of 0.15 and a mean of 0.85, the value of \( \beta_{c} \) stands "typically" between 0.7 and 1. For a non-negligible number of companies/year, the default point indicator is higher than one, something which FP indicate is not in principle rational from the standpoint of the
shareholders. These authors consider that a $\beta_{CDS}$ higher than 1 may be indicative of the fact that the CDS used in the calibration contain components such as cheapest-to-deliver options. Indeed, if the CDS represent an upwardly biased estimate of the credit risk premium in this market, that will translate into a likewise upwardly biased $\beta_{CDS}$. Possibly, however, a $\beta_{CDS}$ higher than 1 is reflecting the presence of factors external to the will of the shareholders when determining the default point (as would be the case of potential liquidity problems). In this respect, the results are consistent with other papers. Davydenko (2005) finds, for example, that for 90% of the companies in default in his sample, the ratio of the market value of the assets to the book value of debt is in the interval (0.27,1.23), very much in line with the content of Fig. 1.

**Price Discovery**

In their study, FP evaluate the different speed with which the bond market, the CDS market and the equity market incorporate fresh information on credit risk. One fundamental conclusion of this paper is that the equity market leads in this respect the CDS (and bond) markets. Although a price discovery analysis of this type is not among the central objectives of this study, the availability of a bigger set of companies for a longer period, on one hand, and the modifications proposed for the estimation of the $ICS_{CDS}$ series, on the other, advise ascertaining to what extent the same result holds in our sample.\(^{12}\)

The analysis is made considering the following VAR model on the daily increases in credit risk premia in both markets;\(^{13}\)

\(^{12}\) Clearly, and compared with the paper by Forte and Peña (2006), our analysis is partly limited by not having information on the bond market. It would not seem, however, that this could significantly affect the conclusions on the price discovery process between the CDS market and the equity market.

\(^{13}\) In the case of $ICS_{CDS}$ series, and for the sake of clarity, we omit the sub-index CDS.
\[
\Delta \text{CDS}_t = a_1 + \sum_{z=1}^{Z} b_{1z} \Delta \text{CDS}_{t-z} + \sum_{z=1}^{Z} c_{1z} \Delta \text{ICS}_{t-z} + e_{1t},
\]

\[
\Delta \text{ICS}_t = a_2 + \sum_{z=1}^{Z} b_{2z} \Delta \text{CDS}_{t-z} + \sum_{z=1}^{Z} c_{2z} \Delta \text{ICS}_{t-z} + e_{2t},
\]

where the optimal number of lags is determined following the Schwarz criterion. The Granger causality test finally helps identify which market incorporates earliest the fresh information in relation to credit risk. The results in Table V confirm the FP conclusion in that the equity market leads the CDS market. The table further indicates that this is true for all the periods and economic areas considered.

**An Econometric Model for \( \beta \)**

Once we have a sample of values for the default point indicator, \( \beta_{CDS}(i,T) \), where \( i = 1, \ldots, 96 \) denotes the company, and \( T = 1, 2, 3, 4 \) the year, we can consider an econometric model in which the variable to be explained is \( \ln(\beta_{CDS}(i,T)) \) (logarithm of \( \beta_{CDS}(i,T) \)), and where the set of explanatory variables excludes any reference to the CDS market. The fundamental aim is to analyse the possible application of this model for determining the default point in the case of companies without information on CDS. The explanatory variables considered are as follows: 14

1. \( \sigma_{\text{END}}(i) \): In their study, FP find that volatility is a key factor when explaining differences in the default point between companies (up to 85\% of the variability of \( \beta_{CDS} \) when this parameter is assumed constant for each company). Greater volatility would specifically mean a lower \( \beta \), something habitually forecast by the structural models with an endogenous default barrier. One fundamental problem when using this variable as a regressor is that, to date, the volatility available to us is that which arises from the process of calibration of

14 The data additional to those already available have been taken from WorldScope.
the $\beta_{\text{CDS}}(i,T)$, $\sigma_{\text{CDS}}(i)$. Since we seek to omit any reference to the CDS market, we should consider an alternative measure of volatility for each company.

One option is to estimate $\sigma(i)$ applying the algorithm described in Section II, but setting the value of $\beta$ irrespective of the data on CDS. Specifically, we can estimate the volatility that would arise from assuming that at each point in time $t$ the shareholders choose the value of $\beta$ optimally. We shall denote this value as $\beta_{\text{END}}(i,t)$, where the term $t$ instead of $T$ indicates that $\beta_{\text{END}}$ will take daily values instead of annual ones. The Appendix demonstrates how this optimal default point indicator for shareholders will be given for each $t$ by the following expression:

$$
\beta_{\text{END}} = \sum_{\tau=1}^{10} \left\{ e^{-\tau r} \left[ p(t) - \frac{c(t)}{r} \right] A(t) + \frac{c(t)}{r} B(t) \right\},
$$

$$
P + \sum_{\tau=1}^{10} p(t) B(t)
$$

Note that $\beta_{\text{END}}(i,t)$ is in turn a function of volatility, whereby each proposed new value for $\sigma_{\text{END}}(i)$ in accordance with the aforementioned algorithm entails the re-estimation of each $\beta_{\text{END}}(i,t)$.

After estimating $\sigma_{\text{END}}(i)$ for $i = 1, \ldots, 96$, we see that the correlation with the $\sigma_{\text{CDS}}(i)$ is 99.98%, i.e. the results in both cases are virtually identical. This indicates that the algorithm proposed generates robust estimators of the volatility in respect of the value assumed for the default point indicator.\footnote{The same conclusions are drawn when making alternative estimates (not provided) of the volatility under other assumptions on the value of $\beta$. This is the case, for instance, of a fixed value of 0.75.} We therefore conclude that it is reasonable to use $\sigma_{\text{END}}(i)$ as a measure of the volatility of the total value of the assets.
In line with the theory and with the results obtained by FP, we would expect to find an inverse relationship between volatility and $\beta_{CDS}$.

$b)$ $r(i,T)$: as a measure of the risk-free rate we will use the year-long average of the swap rate at 5 years. The effect that this variable may have on the default point is ambiguous. On one hand, as the interest rate increases the value of the current debt diminishes, which increases the incentives to repay it (lower $\beta$). On the other, if this increase persists, it will in the long-term entail a higher financing cost, which may more readily dissuade the company from adhering to compliance with its commitments to creditors (higher $\beta$). Situations involving higher interest rates may at the same time increases the influence of liquidity variables, which tends to complicate even further the prediction on the net effect.16

c) $Payout(i,T)$: Defined as interest plus dividends over total assets (book value). A bigger payout would indicate greater capacity on the company's part to generate free cash flows with which to remunerate investors, and in particular creditors. The bigger the payout, therefore, the lower we would expect $\beta_{CDS}$ to be.

d) $Lever(i,T)$: Leverage of company $i$ in year $T$ at market values. This will be approximated by taking total liabilities over total liabilities plus stock market capitalisation. Leverage per se should not prove to be an explanatory variable of $\beta$, since the default point indicator is in itself a standardised measure, due precisely to the level of debt, of the default point. It may however prove to be a good indicator of the presence of relevant variables not included in the

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16 In the case of the regressors $c$ to $g$ described below, the value considered is the average of the figure considered as at 31 December of year $T$ and the figure as at 31 December of year $T$. The reason for applying this procedure (and which motivates also the definition of the variable $r(i,T)$), is that the $\beta_{CDS}(i,T)$ represents an average value for year $T$, and it has in fact been estimated drawing on a linear interpolation between the book figures at the beginning and end of year $T$. Except for regressor $g$, we further consider these variables with a quadratic term in order to capture potential non-linearities.
analysis. Specifically, we might establish that those companies that are most leveraged are so precisely because they are better able (for other reasons) to withstand lower values for their assets without defaulting on debt payment. Our hypothesis will therefore have it that there is an inverse relationship between leverage and $\beta_{CDS}$.

e) $LM2B(i,T)$: Logarithm of the market-to-book ratio (total liabilities plus stock market capitalisation over total book-value assets). The market-to-book ratio may be considered an indicator of the company's opportunities for future growth. In this respect, we would expect an inverse relationship between this ratio and $\beta_{CDS}$.

f) $Size(i,T)$: Company size measured by the logarithm of total assets in thousands of dollars. Size can be relevant for determining $\beta$ for several reasons. Firstly, greater size would allow the company reader access to sources of financing, which would allow it to meet debt commitments and thus put back default. It is the biggest companies, moreover, that will tend to show greater diversity in their creditors, and those which therefore bear greater costs in the event of a hypothetical debt renegotiation. That would likewise lead the company to delay such a process. For both reasons, our expectation would be that the greater the size, the smaller $\beta_{CDS}$ would be.

g) $Quick(i,T)$: Following Davydenko (2005), we introduce the quick ratio (short-term assets over short-term liabilities) as a measure of liquidity. It should be stressed that the consideration of liquidity as a determinant of default would strictly require it to be modelled as an additional state variable. In line with Davydenko (2005), we should define a structural model in which both a decline in the value of the assets to below the default point and a fall in the quick ratio to below a certain critical threshold might result in default. As far as this paper is concerned, we shall simply introduce this measure of liquidity as one more explanatory variable under the assumption that, all other things being equal, the most liquid companies tend to default on the payment
of their debt for lower levels of the total value of their assets (lower $\beta_{CDS}$).

$h)$ $\text{Euro}(i)$: Dummy variable that takes a value of 1 if the company belongs to the euro area and 0 otherwise. The aim of including this variable and the following one is to study the presence of factors specific to the economic areas considered in determining $\beta_{CDS}$.

$i)$ $\text{Yen}(i)$: Dummy variable that takes a value of 1 if the company is Japanese and 0 otherwise.

$j) L\beta_{\text{END}}(i,T)$: Logarithm of the average of $\beta_{\text{END}}$ for company $i$ in year $T$. The inclusion of the endogenous default point as an explanatory variable has a dual purpose. First, to determine to what extent the values that we obtain for $\beta$ further to calibration with the CDS market, $\beta_{CDS}(i,T)$, match those that, theoretically, shareholders would optimally choose. Second, to analyse whether the use of this variable adds explanatory power to a model where variables such as volatility, risk-free rate and payout, which bear on the determination of $\beta_{\text{END}}$, have already been included under the assumption of a linear or quadratic relationship.

**Results**

Table VI shows the correlation between the different variables considered in the analysis, including the dependent variable $L\beta_{CDS}(i,T)$. Table VII gives the results of an initial estimation (REG 1) in which volatility is included as the sole explanatory variable. The coefficient is negative and significant at 1%. This variable, though not reflecting intra-company variability, explains on its own more than 40% of the variability in the logarithm of $\beta_{CDS}$, which confirms its importance as a determinant of the default point.
The following column shows the results of a second estimation (REG 2) which includes the other explanatory variables with the sole exception of $L\beta_{END}$. In this case, volatility retains a negative and highly significant coefficient. The coefficient of the swap rate is positive and significant, which may be interpreted as a sign that, given an increase in interest rates, the prospect of greater future financing costs bears more on the decision to default on debt payments than the loss of value of the current debt. The payout does not prove significant either in the linear or the quadratic term. Leverage, for its part, proves significant in both cases. The value of the coefficients confirms that the logarithm of $\beta_{CDS}$ would be a diminishing function of leverage for whatsoever possible value of this variable. Regarding the market-to-book ratio, an inverse relationship generally holds. The significance of the quadratic term, however, makes this negative relationship conditional on this ratio taking values of less than 1.7, which holds in our sample for around 84% of the observations. The size variables are likewise significant. Their coefficients would indicate that, as forecast, the greater the size, the lower the $\beta_{CDS}$, although such an inverse relationship is conditional upon a total asset value exceeding 2.4 billion dollars. In our case this is true for over 99% of the observations. As regards the quick ratio, the coefficient is negative and significant in line with the starting hypothesis. The coefficient of the dummy variable for the euro area is negative and significant, while the coefficient of the dummy for Japan does not prove significant. These findings would be consistent with a value for $\beta_{CDS}$ in the case of the euro area of around 9% below the value it would take for a similar company in the United States. Finally, mention should be made of the explanatory power of the model, since over 83% of the variability in the logarithm of $\beta_{CDS}$ is accounted for by a very small number of readily accessible variables.

The third column of Table VII contains the results of a third regression (REG 3) in which the logarithm of the average value for the year of $\beta_{END}$ is considered as the only explanatory variable. Both the
constant and the logarithm of $\beta_{END}$ are significant at 1%. It is not possible, moreover, to reject the null hypothesis that this latter coefficient is equal to 1 (t-statistic equal to -0.5180). The significance of the constant indicates, however, that the theoretical optimal default point tends on average to underestimate the true value by around 10%.

The correlation between volatility and $L\beta_{END}$ (Table VI) is -0.94, which suggests that this variable is the key element in the determination of $L\beta_{END}$. If $L\beta_{END}$ reflects the influence of volatility on the default point indicator better than the linear relationship assumption, and if moreover it correctly incorporates the influence of other variables such as the risk-free rate or the payout, then the explanatory power of REG 3 should be greater than that of REG 1. But this does not appear to be the case. The adjusted R2 falls from 41% to 34% on moving from REG 1 to REG 3.

The results of the following estimation (REG 4), in which the logarithm of $\beta_{END}$ has been added as an explanatory variable to the variables included in REG 2, are included in the fourth column of Table VII. Despite the high correlation between volatility and $L\beta_{END}$, the coefficients of both variables are significant and with the expected sign, indicating that their individual effects are accurately captured and that there are no relevant problems of multicollinearity. The adjusted R2 increases slightly relative to REG 2 (0.5%), which likewise supports the idea that both variables contain complementary information on $L\beta_{CDS}$. Also of note is the fact that, relative to REG 2, size in its linear term and the quick ratio cease to be significant, with the remaining variables retaining their significance.

The question arises, once the rest of the variables are included, as to whether volatility continues to be more significant than $\beta_{END}$ for explaining the $\beta_{CDS}$. Column 5 in Table VII repeats the second regression, but omitting volatility and including $L\beta_{END}$. The adjusted R2 falls from 83% to 65%, indicating that in this case volatility
continues to have greater explanatory power than $L\beta_{END}$.

In order to obtain a final model for the estimation of the default point indicator in the absence of information on CDS, we perform a final estimation (REG 6). For this we take REG 4 as a basis, and sequentially eliminate those variables that show least significance. The process ends when the coefficient of all the variables that remain in the model is significant at 10%. One interesting figure is that this process leads the payout coefficient, in its quadratic term, to become negative and for the first time significant, which confirms the inverse relationship that had been expected. The negative relationship between leverage and $\beta_{CDS}$ continues to be confirmed in this regression for any possible value of this variable. Regarding the logarithm of the market-to-book ratio, the negative relationship that was previously conditional upon a value for this ratio below 1.7 is associated now with a value below 1.78, which is observed in our sample for approximately 90% of the observations. Finally, the negative effect of size is now verified irrespective of the value this variable takes. The final adjusted R2 is around 84%.

An important consideration is that though we have treated the $\beta_{CDS}$ as a cross-section sample, these values represent actually an incomplete panel sample. If there are unexplained individual effects in the relationship, then the estimation by ordinary least squares (OLS) used so far might prove inappropriate.

The presence of individual heterogeneity is verified by means of a decomposition of the variance of the errors, with the null hypothesis of absence of individual effects being rejected at 99% for all the regressions in Table VII. For this reason we repeat the estimates in this table, but this time applying a panel regression with random effects through feasible generalised least squares (FGLS). The panel analysis with fixed effects is not considered since the ultimate objective is to apply the regression to out-of-sample companies. The results are in Table VIII. Of note are two essential differences in
respect of the conclusions of Table VII. The first is that now the $\beta_{END}$ appear slightly more significant for explaining changes in the $\beta_{CDS}$ than volatility (REG 3 versus REG 1, and REG 5 versus REG 2). Unlike what happened with the cross-section regressions, this would indicate that the effect of volatility on $\beta_{CDS}$ is better reflected through its effect on $\beta_{END}$ than through a linear relationship. Nonetheless, the coefficients of both variables are once again significant and with the expected sign in the overall regression, with the elimination of either of them (REG 2 y REG 5) giving rise to a reduction in the adjusted R2. It therefore appears that in this case too they offer complementary information on the default point indicator. The second difference is that size ceases to be a significant variable. Regarding the other variables that remained in the final regression by OLS, such variables also remain in the panel regression with random effects, and with an identical sign. The coefficients of leverage in its linear and quadratic term are, as before, consistent with a negative relationship to $\beta_{CDS}$ for any possible value of this variable. The negative relationship between the market-to-book ratio and $\beta_{CDS}$, and which in the OLS regression was ultimately conditional upon a value for this ratio of less than 1.78, now holds for market-to-book ratio values of below 1.95, which in our sample holds in approximately 95% of the observations. The adjusted R2 of REG 6 for the panel estimation with random effects is 95%.

**Usefulness of the Econometric Model**

**A. Out-of-sample estimation of $\beta$.**

A key question is whether the econometric model obtained is applicable to out-of-sample companies, since that would allow the credit risk premium in the equity markets to be estimated, even in the absence of information on CDS. In this respect, we should ask whether, with a view to an out-of-sample estimation, the panel model
with random effects - in which part of the individual variability may be explained randomly - is advisable, or whether on the contrary the cross-section model is preferable, where it is sought to include all the individual heterogeneity via the explanatory variables.

In order to test which procedure is preferable, we begin with an out-of-sample estimates of the $\beta(i,T)$ based on the cross-section econometric model. To do this we randomly divide the sample into six groups with 16 companies each. We then re-estimate the REG 6 model, eliminating the companies from the first group, and we apply the results to predict the $\beta(i,T)$ of the companies of the excluded group. Repeating this procedure for the other five groups, we finally obtain an out-of-sample estimate of the default point indicators for each company-year based on the cross-section econometric model. We denote these estimated values as $\beta_{\text{REG}}(i,T)$. An identical procedure is then applied to the panel model with random effects (the same 6 groups), giving rise to an alternative estimate of the $\beta(i,T)$, which in this case we denote $\beta_{\text{PAN}}(i,T)$.

Table IX compares the errors of each type of estimate. Evidently, although the average estimation error is close to zero for both procedures, the cross-section model proves more accurate (lower standard deviation and lower standard deviation in absolute values). These results would support the idea that, with a view to an out-of-sample estimate, the cross-section model is preferable.

**B. Estimation of ICS without information on CDS.**

Once the $\beta_{\text{REG}}$ have been estimated, it is possible to deduce the series of credit risk premia drawing on the equity market, the result of assuming such values for the default point indicator ($ICS_{\text{REG}}$). The aim would be to test to what extent the out-of-sample use of the econometric model produces results consistent with those derived
from the use of the $\beta_{\text{CDS}}$ ($ICS_{\text{CDS}}$), and therefore whether this model may be useful for estimating credit risk premia in the equity market in the case of companies without CDS. At the same time, it will be worthwhile studying whether the application of the econometric model is an improvement on alternative methods for determining the default point. The first alternative will be to consider the series generated from the $\beta_{\text{END}}$ ($ICS_{\text{END}}$). Second will be the series resulting from setting for each $t$ the value of $\beta$ in keeping with the procedure followed by Moody’s-KMV. Specifically, we shall define $\beta_{\text{KMV}}$ as the ratio of short-term liabilities plus half of the long-term liabilities over total liabilities (resulting series $ICS_{\text{KMV}}$). We consider below the alternative of setting a constant value for $\beta$. In this respect, we shall analyse the results of imposing $\beta$ equals 0.75 for any period or company. This value, which we shall denote $\beta_{0.75}$ (series $ICS_{0.75}$), would be in line with the average values obtained by Leland (2004) and FP. Moreover, we will study the results arising from assuming that $\beta$ equals 0.85 ($\beta_{0.85}$ series $ICS_{0.85}$). We hereby seek to test to what extent the variability of $\beta$ included in $\beta_{\text{REG}}$, improves the estimates in respect of assuming a constant value equivalent to the average. Finally, we shall impose $\beta$ equals 1 ($\beta_{1.00}$ series $ICS_{1.00}$), which is equivalent to assuming that default arises when the value of the assets falls to the face value of the debt.

Table X shows the differentials with respect to the CDS series in terms of the procedure selected to set the value of $\beta$. The main conclusions would be as follows:

1. The $ICS_{\text{REG}}$ tend to overestimate the CDS (by 41.05% on average) to a greater extent than the $ICS_{\text{CDS}}$ (7.93%). This is despite the fact that the differential between $\beta_{\text{REG}}$ and $\beta_{\text{CDS}}$ is on average equal to zero. These results stem from the non-linear

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17 See Crouhy et al. (2000). It is worth specifying that this exercise in no case seeks to replicate the type of results produced by the Moody’s-KMV methodology.
relationship between the premium estimated in the equity market and $\beta$. Specifically, an overestimation of $\beta$ appears to produce a bigger error in absolute values than the error generated by an underestimation of $\beta$ on the same scale. The presence of positive extreme values in the differential between the $ICS_{REG}$ series and the CDS series implies, moreover, that the average is of such differences is far above the medians, and that the standard deviation is also very high. Thus, for example, the average absolute basis ($avab$) is 57.41 bp, practically double the median of this statistic which is 30.65 bp. The standard deviation is, moreover, 110.03 bp. If the three companies with the biggest $avab$ are eliminated, the average draws close to the median (39.56 bp) and the standard deviation falls to 33.81 bp.

2. Regarding the $\beta_{END}$, the underestimation of the $\beta_{CDS}$ tends to give rise, as expected, to an underestimation of the CDS, although the differential is in many cases lower than that resulting from use of the $\beta_{REG}$. Thus, for instance, the $avab$ is on average 47.80 bp (compared with 57.4 bp previously). If, in contrast, the medians are compared instead of the averages so that the extreme values have less weight, the $ICS_{REG}$ slightly outperform the $ICS_{END}$ as predictors of the CDS. The greater penalisation associated with the potential overestimation of $\beta$, and which gives rise to extreme errors, would once again justify the results. The $\beta_{REG}$ are specifically a better approximation to the true value of $\beta$; however, the tendency of the $\beta_{END}$ to underestimation enables them to avoid extreme errors in the prediction of the CDS, prompting better results on average.

3. In this same respect, the use of the $\beta_{KMV}$ tends to produce an underestimation of the CDS on a similar scale to that generated by the $\beta_{END}$. The dispersion of the differentials is, however, greater in the first instance. Thus, while the average of the $avb$ is
around -20 bp for both procedures, the standard deviation for the $ICS_{KMV}$ (93.94 bp) is greater than for the $ICS_{END}$ (59.69 bp). The average of the $\beta_{END}$ is 0.75, similar to that of the $\beta_{K MV}$, which is 0.73. The tendency of both procedures to underestimate $\beta$ produces a negative bias in the estimation of the credit risk premia. The lesser dispersion of the differential that arises on using the $\beta_{END}$ shows that, despite the error, this procedure captures better the variability of $\beta$ than $\beta_{K MV}$.

4. Using a fixed value of 0.75, close to the average of the $\beta_{END}$ and of the $\beta_{K MV}$, produces an $avb$ which is on average similar to that derived from applying these other procedures (around -20 bp). Once again, the greater capacity of the $\beta_{END}$ to represent the variability of $\beta$ confers greater accuracy upon it despite the error (standard deviation of 59.69 bp compared with 84.42 bp on applying $\beta_{0.75}$). On comparing the dispersion of the differentials, this constant value appears to produce better results than $\beta_{K MV}$. In other words, the definition of the default point as short-term liabilities plus half of the long-term liabilities does not provide in this model explanatory power in relation to the variability of $\beta$, actually worsening the results in respect of the assumption of a constant value similar to the average.

5. Although 0.85 represents the average of the distribution of the $\beta_{CDS}$, setting its value generally produces worse results than setting 0.75. Once again, the explanation is to be found in the high non-linearity of the relationship between the ICS and $\beta$. With $\beta_{0.85}$, underestimations and overestimations are made practically in equal proportions. With $\beta_{0.75}$, underestimations are made to a greater extent, giving rise on average to lower error.

6. Considering that the company defaults when the value of its assets falls below the face value of the debt ($\beta_{1.00}$) ultimately
means a clear tendency to overestimation of the CDS. The high penalisation in terms of error associated with the overestimation \( \beta \) means that, among all the procedures, this is the one which gives the worst results.

It can be deduced from the foregoing that the econometric model is capable of reflecting much of the variability of \( \beta \) (up to 84% within the sample). The high sensitivity of the ICS to the value assumed for \( \beta \), and the special penalisation associated with an error of overestimation of this parameter mean that, although in median terms the \( \beta_{\text{REG}} \) produce better results, on average greater errors may be committed than with other methods. The prediction of the exact value of the credit risk premium generated from the model should thus be viewed with caution, and its use may be more appropriate for establishing credit risk ranges than for a strict valuation. This is what Table XI, Panel A suggests. This Table shows the distribution of CDS by range of values. It can be seen how, among the 62,571 observations, more than 50% are below 50 bp, and practically 80% below 100 bp. The Table also indicates the proportion of correctness for each method and range. Thus, for example, on 86% of the occasions on which the CDS are in the range \([0.50]\), the ICS_{CDS} are also in that range. The use of the \( \beta_{\text{REG}} \) exceeds any other method (excluding \( \beta_{\text{CDS}} \)) in terms of expected correctness for the range \((50.500]\). It is surpassed, however, by the methods \( \beta_{\text{END}} \) and \( \beta_{0.75} \) for the range \([0.50]\), and by the methods \( \beta_{0.85} \) and \( \beta_{1.00} \) for the range >500.

We thus see how those procedures that tend to underestimate the premiums are correct to a greater extent in respect of the range that concentrates a bigger percentage of observations. Expressed otherwise, as most of the CDS represent values below 100 bp, the procedures that systematically predicted low premiums (\( \beta_{\text{END}} \) and \( \beta_{0.75} \)) tend to be correct to a greater extent than the procedure of applying \( \beta_{\text{REG}} \), since the latter seeks to distinguish between companies with low premiums and companies with high premiums.
In order to assess more formally the discriminatory capacity of each method, the following null hypothesis may be considered:

\[
H_0 : CDS(i,t) \leq 150
\]  

(14)

The value of 150 bp would be in the interval between 87.9 bp and 269.5 bp that Houweling and Vorst (2001) find on average for companies rated BBB and BB, respectively. The interpretation of the null hypothesis might thus be that the company retains the investment rating according to the CDS market agents.

Table XI, Panel B, shows the probability of error type I (eI) and error type II (eII) in terms of the method applied for determining \( \beta \). In the case of the direct estimation with CDS (\( \beta_{CDS} \)), the probability of eI is 4% and that of eII is 14%. These values rise to 9% and 31%, respectively, on applying the econometric model (\( \beta_{REG} \)). While the underestimation of premiums associated with the use of \( \beta_{END} \), \( \beta_{0.75} \) and even \( \beta_{KMV} \) holds the probability of eI below 5%, the probability of eII is in all cases above 65%, which indicates scant testing power. The opposite case would be that of \( \beta_{1.00} \). On overestimating the premiums, the probability of eII falls to 9%, but significance worsens considerably (the probability of eI climbs to 42%). Mid-way between the results of applying \( \beta_{0.75} \) or \( \beta_{1.00} \) would be the results for \( \beta_{0.85} \), with a level of significance of 12% and a probability of eII of 51%. This is possibly the case where the usefulness of \( \beta_{REG} \) is most evident, since both types of errors lessen on applying a method that takes into account not only the average of \( \beta \) (0.85), but also the variability around that average. In sum, of all the procedures that do not require direct observation of the CDS, the use of \( \beta_{REG} \) is that which, maintaining a level of significance below 10%, entails greater testing power. This power would specifically be twice that associated with the best possible alternative \( \beta_{END} \) (69% as opposed to 35%).
Conclusions

In this paper we have considered a broad sample of US, European and Japanese companies during the period 2001-2004, calibrating the default barrier for each company-year on the basis of their CDS premia. Although the procedure used has broadly been that described by Forte and Peña (2006), two fundamental contributions should be highlighted. Firstly, we calibrated not only the default point, but also bankruptcy costs (exogenous in the original model). To do this we adjusted the mean recovery rate forecast for each sector to its historical average. Secondly, we constructed an econometric model which allows the default point indicators to be estimated without resorting to information on the CDS market. The model, which explains up to 84% of the variability of the default point indicators within the sample, uses only information on the equity market and a small number of accounting items. The main advantage is thus its potential application to companies for which no data on CDS are available. Compared with other alternatives for setting the default point when this information cannot be had (the optimal default point for shareholders, the default point in the Moody’s-KMV model and various inter-company constant default point options), the use of the econometric model significantly enhances the capacity for differentiating between companies with an investment grade rating and companies with a non-investment grade rating. Specifically, faced with the null hypothesis that a company’s CDS is less than 150 bp (it has an investment grade rating), the use of the econometric model maximises testing power maintaining a level of significance below 10%. This power is of the order of 69%, double that of the best possible alternative, consisting of setting the default point to the shareholders’ optimal point.
Appendix

**Endogenous β**

The endogenous default point is determined according to the smooth-pasting condition:\(^{18}\)

\[
\frac{\partial S(V, t)}{\partial V} \bigg|_{V = V_{a}} = 1 - \frac{\partial D(V, t|\alpha = 0)}{\partial V} \bigg|_{V = V_{a}} = 0 \quad (A.1)
\]

Starting from expression (2) for the value of each bond it is possible to resolve (A.1) and obtain the optimal default barrier

\[
V_{B, END} = \frac{\sum_{i=1}^{N} \left\{ e^{-r_{i}} \left[ p(\tau_{i}) - \frac{c(\tau_{i})}{r} \right] A(\tau_{i}) + \frac{c(\tau_{i})}{r} B(\tau_{i}) \right\}}{1 + \sum_{i=1}^{N} \rho(\tau_{i}) B(\tau_{i})} \quad (A.2)
\]

where

\[
A(\tau_{i}) = \frac{2f\left(a\sigma_{\sqrt{\tau_{i}}}\right)}{\sigma_{\sqrt{\tau_{i}}}} + 2aN\left(a\sigma_{\sqrt{\tau_{i}}}\right) \quad (A.3)
\]

\[
B(\tau_{i}) = (a-z)N\left(-z\sigma_{\sqrt{\tau_{i}}}\right) + (a+z)N\left(z\sigma_{\sqrt{\tau_{i}}}\right) + \frac{2f\left(z\sigma_{\sqrt{\tau_{i}}}\right)}{\sigma_{\sqrt{\tau_{i}}}} \quad (A.4)
\]

The constraint \(\alpha = 0\) is assured by setting in turn the following condition on the coefficients \(\rho(\tau_{i})\)

\[
\sum_{i=1}^{N} \rho(\tau_{i}) = 1 \quad (A.5)
\]

---

which is met imposing

$$\rho(\tau_i) = \frac{p(\tau_i)}{P}; \quad i = 1, \ldots, N \quad (A.6)$$

For its part, the endogenous default point indicator will be simply

$$\beta_{\text{END}} = \frac{V_{B,\text{END}}}{P} \quad (A.7)$$
References


